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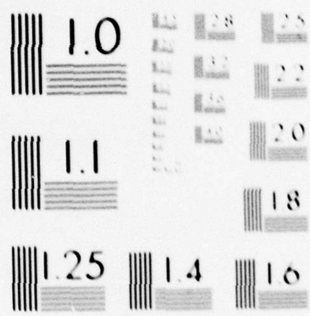

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DECISION CONTROL MODELS IN OPERATIONS RESEARCH

Final Report

Harvey M. Wagner, Principal Investigator
Richard Ehrhardt, Co-principal Investigator

March 6, 1978

U.S. Army Research Office

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The University of North Carolina at Chapel Hill
Chapel Hill, N.C. 27514

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The research concentrated on one major topic of investigation: statistical properties of inventory models. The analysis deals with comparing the operating characteristics of several important stockage rules when the underlying demand distribution parameters are estimated from a limited sample of historical data (~~say,~~ 13, 26 or 52 observations). Particular attention is given to the frequently observed case where the probability of zero demand occurring within any time period is significantly large (~~say~~, half of the

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20. historical observations indicate zero demand), but the positive levels of demand are not necessarily small. The overall context of the analyses is a large-scale inventory system containing thousands of individual items, and thus, the computational burden of implementing the stockage policies must be recognized in the comparison of different stockage rules. The research approach encompasses applied Markovian probability analysis and computer simulation analysis containing sophisticated statistical autoregressive time series methods.

FOREWORD

The research concentrated on one major topic of investigation: statistical properties of inventory models. The analysis deals with comparing the operating characteristics of several important stockage rules when the underlying demand distribution parameters are estimated from a limited sample of historical data (say, 13, 26, or 52 observations). Particular attention is given to the frequently observed case where the probability of zero demand occurring within any time period is significantly large (say, half of the historical observations indicate zero demand), but the positive levels of demand are not necessarily small. The overall context of the analyses is a large-scale inventory system containing thousands of individual items, and thus, the computational burden of implementing the stockage policies must be recognized in the comparison of different stockage rules. The research approach encompasses applied Markovian probability analysis and computer simulation analysis containing sophisticated statistical autoregressive time series methods.

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DECISION CONTROL MODELS IN OPERATIONS RESEARCH

1. Background of the Inventory Research Program

Over the past two decades, mathematical analysis of inventory stockage models has made great progress, and real-life implementation of inventory systems based, at least in part, on the results of this modern research has taken place. But typically practitioners have applied theoretically derived formulas to actual situations in which the model's underlying assumptions are far from being met. A critical divergence between the model's assumptions and the reality occurs with regard to the amount of knowledge the systems' designer has about the underlying demand distributions. With rare exceptions (some of which are cited in the References), theoretical inventory models assume that the probability distribution of an item's demand is completely specifiable. In the most sophisticated theoretical inventory models, heavy reliance is placed on this assumption in deriving an algorithm for computing an optimal policy (1, 15, 20, 25, 26, 27). Many other theoretical analyses (2, 10, 12, 21, 28) make gross assumptions that permit stating inventory formulas that require only a few parameters of the demand distribution; but even then, these approximate analyses assume that the small number of parameters are known with certainty. When practitioners utilize such models, they typically employ standard statistical procedures to estimate the parameters in the approximation formulas, and pay little if any attention to modifying the estimates due to the special nature of the application. At present, there is a significant knowledge gap concerning the nature of the systematic performance and prediction biases that are introduced when these statistical estimates are substituted for parameters in inventory models.

The seriousness of the problem stems in part from the fact that in most real situations, only a very limited amount of past data on demand is available, and frequently such data contain a large proportion of zero-demand observations. For example, in a 26 week history of demand for an item, perhaps no demand occurred during 10 of the weeks, and in the remaining 16 weeks, the demand values displayed wide variation (such as, 30, 100, 7, 245, etc.). We currently have no scientific understanding of what stockage rules perform acceptably well in such an environment, and how much accuracy we have in our estimates of future system performance based on the past data. The following scenario will clarify further the context for the work currently under support.

Consider an inventory manager who must design a system of replenishment rules for the stockage of possibly thousands of items. Assume that, without too much difficulty, the manager can specify the parameters of the criterion function, so that there is a direct way of determining whether one system is better than another, provided that the relevant associated operating characteristics for each system can be obtained. For example, assume that the manager knows the costs of holding inventory, making replenishment decisions, reviewing the stock status of an item, etc. Also assume that the manager is able to articulate the objectives or costs relating to stockouts; to illustrate, the manager may specify a cost attached to a unit of demand that cannot be filled, or perhaps the manager

1. Background of the Inventory Research Program (continued)

may set a limit on the probability of a stockout, or possibly target fulfilling a certain fraction of total demand.

In designing the system, the manager must select a class of decision rules, and preferably, pick optimal policies from the selected class. To illustrate, the complete economic preference function of the manager may imply that if the probability distribution of demand is known, then an optimal policy is of an (s,S) form: when inventory on hand plus on order falls below s, place an order so that, as a consequence, inventory on hand plus on order equals S. The research of Veinott and Wagner (1) shows how to compute optimal values for (s,S) in this circumstance, and the research of Wagner, O'Hagan, and Lundh (2) tests out various numerical approximations that, since the publication of that paper, have turned out to be implementable in practice. But if the manager faces considerable uncertainty about each item's *demand distribution*, because there is only a limited number of past observations on demand, it is by no means obvious how to proceed. In fact, the decision process for systems design becomes much more complex than the mere selection of parameter values for (s,S).

First, the manager must decide how much past demand data to actually use. If the manager suspects that the underlying forces causing the demand for an item shift from time to time, the manager may want to ignore data that are older than some number of periods, which then must be specified. Since the system is to operate over an indefinite future, the manager also must determine how often to update the rules, that is, how often to discard old historical data and recompute the replenishment policies' parameters. For example, the manager may choose to revise the policies every six months and, at each revision, utilize the past six months' data.

Second, since the manager does not know the form of the demand distribution, the manager must either guess at the form, then statistically estimate the distribution's parameters for each item (possibly using a prior distribution), and finally compute policies assuming that the estimated distributions are in fact the true distributions; or the manager may choose to use an "approximately optimal" form for the replenishment *policy* (such as let s equal a number of mean demands plus a multiple of the standard deviation of demand, and $S=s+D$, where D is the familiar Wilson square-root lot size formula that requires knowing only the mean demand), and then select statistical estimators for the parameters required by this policy form. This enumeration does not make apparent the full design decision possibilities because, within each option, the manager has many alternatives from which to choose. To complete the scenario, suppose that the manager does make a particular selection from among all these many options.

Third, the manager must decide *before* implementing the system whether the design parameters are well set. For example, if the designer sets the

1. Background of the Inventory Research Program (continued)

reorder level $s = a \cdot (\text{mean demand}) + b \cdot (\text{standard deviation of demand})$, then appropriate values for a and b must be chosen. Since the manager may realize that several approximations have been compounded in the design of rules, the manager may hesitate to rely solely on theoretical probability distribution analysis that is based on complete knowledge of the demand distribution. Most likely, the manager will employ retrospective simulation, that is, the manager will reuse the limited amount of past data from which the mean demand and the standard deviation of demand were estimated to adjust the values of a and b so that the resultant *statistical estimate* of the criterion function is at an optimal value. (Actually, the manager probably will set the same values for a and b over a large number of items, and so will perform the adjustment process via retrospective simulation by using an aggregate objective function.) The manager also will estimate several operating characteristics of importance, such as average inventory on hand, the probability of a stockout, the fraction of demand filled, etc.

To summarize the scenario, we see that the system's designer must select in concert the number of historical observations to use, the frequency for repeating the reestimation process, the form of the replenishment rule, the statistical estimators to produce the demand parameters required by the rule, and the design parameters of the rule. The manager makes all of these choices based at least in part, on simulating how the proposed system would have performed in the past (and in doing so, typically uses the same limited data for both estimation and prediction). In all the discussion to follow, we employ the term *system's design* to mean the entire composite of these many choices.

The nature of the currently supported research program is to examine

1. How good are the manager's statistical estimates of the system's future performance for selected choices of the system's design? Similarly, how good are theoretically based estimates of future performance?

2. Under what circumstances do certain "approximately optimal" policies perform better than others? Are some policies statistically more robust than others?

3. How do the statistical estimates of the system's future performance, as well as the performance characteristics themselves, depend on choice parameters such as the number of historical observations to use and the frequency of reestimating the policy's demand parameters?

Special attention is to be given to systems in which the demand distributions display a high frequency of zero demand (the positive levels of demand, however, need not be anywhere near zero).

Except under extremely special assumptions, it is not possible to derive computationally practical formulas for the exact statistical

1. Background of the Inventory Research Program (continued)

distributions associated with the above questions. The best hope for increasing our ability to answer the research questions above in a wide variety of situations is to derive more workable numerical approximations. With that objective in mind, we have built under the previous and present research grants a computer simulation model that yields the actual operating characteristics and the corresponding properties of statistical estimators of these operating characteristics for inventory systems that contain uncertainty about the demand distributions.

2. Summary of Results

This section summarizes the results that have been obtained. A detailed account of the results is contained in the following reports.

MacCormick, A. (1975), Statistical Problems in Inventory Control, ARO and ONR Technical Report 2, December 1974, School of Organization and Management, Yale University, 244 pp.

Estey, A.S. and R.L. Kaufman (1975), Multi Item Inventory System Policies Using Statistical Estimates: Negative Binomial Demands (Variance/Mean = 1), ARO and ONR Technical Report 3, September 1975, School of Organization and Management, Yale University, 85 pp.

Ehrhardt, R. (1975), Variance Reduction Techniques for an Inventory Simulation, ARO and ONR Technical Report 4, September 1975, School of Organization and Management, Yale University, 24 pp.

Kaufman, R. (1976), Computer Programs for (s,S) Policies under Independent or Filtered Demands, ARO and ONR Technical Report 5, School of Organization and Management, Yale University, 65 pp.

Kaufman, R. and J. Klincewicz (1976), Multi-Item Inventory System Policies Using Statistical Estimates: Sporadic Demands (Variance/Mean = 9), ARO and ONR Technical Report 6, School of Organization and Management, Yale University, 58 pp.

Ehrhardt, R. (1976), The Power Approximation: Inventory Policies Based on Limited Demand Information, ARO and ONR Technical Report 7, School of Organization and Management, Yale University, 58 pp.

Klincewicz, J.G. (1976), Biased Variance Estimators for Statistical Inventory Policies, ARO and ONR Technical Report 8, School of Organization and Management, Yale University, 24 pp.

2. Summary of Results (continued)

Klincewicz, J.G. (1976), Inventory Control Using Statistical Estimates: The Power Approximation and Sporadic Demands (Variance/Mean = 9), ARO and ONR Technical Report 9, School of Organization and Management, Yale University, 52 pp.

Klincewicz, J.G. (1976), The Power Approximation: Control of Multi-item Inventory Systems with Constant Standard-deviation-to-mean Ratio for Demand, ARO and ONR Technical Report 10, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 47 pp.

Kaufman, R.L. (1977), (s,S) Inventory Policies in a Nonstationary Demand Environment, ARO and ONR Technical Report 11, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 155 pp.

Ehrhardt, R. (1977), Operating Characteristic Approximations for the Analysis of (s,S) Inventory Systems, ARO and ONR Technical Report 12, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 109 pp.

Schultz, C., R. Ehrhardt, and A. MacCormick (1977), Forecasting Operating Characteristics of (s,S) Inventory Systems, ARO and ONR Technical Report 13, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 73 pp.

Software Development. Several useful computer programs have been constructed to assist in the research. Using the method in Veinott and Wagner (27), one program calculates optimal infinite horizon (s,S) policies. The routine accepts as demand distributions, Poisson, Negative Binomial, and compound Negative Binomial. It not only computes an optimal policy, but also calculates the associated expected cost, set-up cost, holding cost, penalty cost, replenishment frequency, and stockout frequency. This program also will calculate these same operating characteristics for an arbitrarily specified (s,S) policy (such as an approximately optimal policy).

A second program that is available accepts a stream of actual demands, and computes an optimal (s,S) policy for that stream. The methodology is analogous to that used in the above routine. This program facilitates looking at situations where the demands need not be identically and independently

2. Summary of Results (continued)

distributed. For example, the demands may result from (s,S) replenishment policies being used by the organization's customers, and the computer program includes a subroutine that can generate demands in this fashion.

A simulation model was constructed to analyze statistical policies. The program was originally written to simulate the base-case system and has since been modified to study nonstationary (cyclic) demand distributions. Both simulation programs employ variance reduction techniques and sophisticated statistical analysis of output data. Each simulation tabulates point estimates and standard errors for average period-end inventory, stockout quantity, replenishment frequency, and total cost per period. For the statistical policies, the simulations also calculate forecasts of these quantities at each policy revision.

Base Case Studies. Each study utilizes multi-item systems comprised of 72 items representing for each demand distribution (Poisson, Negative Binomial with variance-to-mean ratio 3, Negative Binomial with variance-to-mean ratio 9) a full factorial design using the parameters specified in Table I.

TABLE I

System Parameters

Factor	Levels	Number of Levels
Demand distribution	Poisson, Negative Binomial (σ^2/μ = 3, 9)	1
Mean demand	2, 4, 8, 16	4
Unit holding cost	1	1
Unit backlog penalty cost	4, 9, 99	3
Replenishment setup cost	32, 64	2
Replenishment leadtime	0, 2, 4	3

2. Summary of Results (continued)

The initial phase of base-case research focused on the Normal Approximation, an approximately optimal policy depending on only the mean and variance of demand. A new policy rule, the Power Approximation, was then derived using asymptotic theory and regression analysis. Table II summarizes the performance of these policies in several multi-item systems. Note that both policy rules are quite close to optimality in these systems. The Power Approximation outperforms the Normal Approximation quite consistently with the greatest differences occurring in items with either high demand variance or large unit penalty cost.

Table III shows how the policy rules perform when statistical estimates are used in place of the actual demand means and variances. The figures displayed in Table III are absolute increases or decreases in cost components over optimal values with percentage differences in parentheses. The best overall results are obtained with the Power Approximation. For example, when the Statistical Power Approximation was computed using 26 periods of data, the average total cost per period for the entire system rises 5.1% when variance/mean = 3, and 11.5% when variance/mean = 9. If as many as 52 periods of data are used, then the degradation is only 6.3% when variance/mean = 9. Realizing that if we were to define "optimal policies" for an environment of partial information about the underlying demand distribution, some degradation over optimal (s,S) policies would occur, these results are encouraging. That is, controlling an inventory system using approximately optimal formula driven by statistical estimates from a limited history of demand in many cases ought to be satisfactory. A notable exception to this generalization arises when the penalty cost is high (that is, the targeted stockout probability is low, of the order .01). For example when a demand history of 26 periods is utilized, a 18% degradation occurs when variance/mean = 9.

Several studies focused on the situation where the probability of zero demand occurring within any time period was significantly large, but the positive levels of demand were not necessarily small. The performance of the Power Approximation and the Normal Approximation were examined. The Power Approximation yielded lower expected total costs than the Normal Approximation under conditions of full information. When statistical information was used to set (s,S) policies, however, the relative performance of the Power Approximation and the Normal Approximation was observed to be a function of mean demand: for items with low demand (μ=8,16), the Normal Approximation was preferred.

The potential for improving statistically-derived policies was examined in a study of the use of biased estimates of the demand variance. Significant savings were found only for inventory items with small mean demand (μ=2) and large unit penalty cost (p/h=99). For these items, total cost was reduced by approximately 4% when the variance estimator was biased upward.

TABLE II

Average Costs per Period for a 72-Item System:
Comparison of Optimal and Approximately Optimal Control

COST COMPONENT	VARIANCE/MEAN = 9 (NEGATIVE BINOMIAL DEMANDS)		VARIANCE/MEAN = 3 (NEGATIVE BINOMIAL DEMANDS)		VARIANCE/MEAN = 1 (POISSON DEMANDS)	
	POWER APPROXIMATION	NORMAL APPROXIMATION	POWER APPROXIMATION	NORMAL APPROXIMATION	POWER APPROXIMATION	NORMAL APPROXIMATION
INVENTORY	26 (1.4)	-115 (-5.9)	-13 (-1.0)	43 (3.5)	13 (1.4)	58 (6.0)
BACKLOG	-18 (-2.7)	161 (23.5)	17 (5.0)	-42 (-12.3)	-9 (-4.6)	-21 (-10.7)
REPLENISHMENT	1 (0.2)	53 (8.6)	-1 (-0.1)	19 (2.6)	3 (0.4)	-24 (3.0)
TOTAL	9 (0.3)	99 (3.1)	3 (0.1)	21 (0.9)	7 (0.4)	12 (0.6)

Note: This table shows the absolute increase or decrease in the cost components over optimal values with percentage differences in parentheses.

TABLE III

Average Costs per Period for a 72-Item Negative Binomial System: Comparison of Statistical Control with Optimal Control Given Full Information

COST COMPONENT	VARIANCE/MEAN = 9 Power Approximation		VARIANCE/MEAN = 9 Normal Approximation		VARIANCE/MEAN = 3 Power Approximation		VARIANCE/MEAN = 3 Normal Approximation	
(a) Revision Interval 13 Periods, Revision History Length 13 Periods								
INVENTORY	121	(6.2)	-61	(-3.2)	*		17	(1.3)
BACKLOG	529	(77.6)	880	(129.0)	*		221	(64.4)
REPLENISHMENT	-3	(-0.4)	44	(7.0)	*		16	(2.2)
TOTAL	647	(20.0)	862	(26.6)	*		253	(10.8)
(b) Revision Interval 26 Periods, Revision History Length 26 Periods								
INVENTORY	54	(2.8)	-118	(-6.1)	22	(1.7)	1	(0.1)
BACKLOG	311	(45.7)	597	(87.5)	94	(27.4)	114	(33.2)
REPLENISHMENT	8	(1.3)	56	(8.9)	4	(0.6)	22	(3.0)
TOTAL	373	(11.5)	535	(16.5)	120	(5.1)	135	(5.8)
(c) Revision Interval 52 Periods, Revision History Length 52 Periods								
INVENTORY	43	(2.2)	-131	(-6.8)	*		-5	(-0.4)
BACKLOG	154	(22.6)	398	(58.4)	*		58	(16.8)
REPLENISHMENT	7	(1.2)	57	(9.1)	*		22	(2.9)
TOTAL	204	(6.3)	324	(10.0)	*		74	(3.2)

Note: This table shows the absolute increase or decrease in the cost components over optimal values with percentage differences in parentheses.

* : Not available.

2. Summary of Results (continued)

Operating Characteristic Analyses. An in-depth analysis of the operating characteristics of optimal, approximately-optimal and statistical policies was conducted. The characteristics under consideration were the period-average values of holding cost, backlog cost, backlog frequency, replenishment cost and total cost. Approximate expressions for the characteristics were derived analytically from theoretical considerations, and from empirical observations. The expressions were generalized and their parameters fit to a large number of observed values of the characteristics using least-squares regression. The resulting approximations are typically within a few percent of the observed values. Such approximations allow, through interpolation, the calculation of operating characteristics for a wide setting of parameter values, given a class of probability distributions. They permit a more accurate determination of parameter snesitivity (including the economic parameters, shape of the demand distribution, and amount of statistical information available). Further, they can yield answers concerning multi-location inventory systems--to illustrate, "What happens to total cost and its components if an item is stocked in a single location rather than in several?"

Nonstationary Demand. A detailed analysis of the nature of optimal policies in a nonstationary environment was undertaken, paying particular attention to the situation in which demand distributions are independent, but the mean demand varies in a cyclic manner. The analysis resulted in a successful adaptation of the Power Approximation to nonstationary environments where the mean and variance of demand are known for each period in the cycle.

Throughout the study, a sixteen-item system was examined for each of five demand models. In all five models, the mean demand was varied in a cyclic manner while the variance-to-mean ratio was held constant at 3. Mean demands were varied by a factor of 3 in Models I and II, and by a factor of 5 in Models III and IV. The fifth model was the base-case assumption of iid demands, and was used as a point of comparison.

Results are summarized in Table IV where Power Approximation costs in a 16-item system are compared with optimal costs for the four cyclic demand models and the stationary model. Notice that the total cost of the approximately-optimal policy is typically within a few percent of the optimal value.

Computer simulation was used to examine the situation in which only a limited number of past demands are known. Each period's demand mean and variance were estimated using regression analysis, and the estimates were used in place of actual means and variances in the computation of modified Power Approximation policies. Results are summarized in Table V, where costs under statistical control are compared with optimal costs, for the four nonstationary models and the stationary model.

2. Summary of Results (continued)

TABLE IV

Average Costs Per Period for a Multi-Item System:
Comparison of Optimal and Approximately Optimal Control
for Several Cyclic Demand Models

COST COMPONENT	STATIONARY MODEL	MODEL I	MODEL II	MODEL III	MODEL IV
INVENTORY	1 (.1)	12 (2.9)	5 (1.1)	37 (8.7)	27 (6.3)
BACKLOG	0 (-.2)	-1 (-1.0)	0 (-.2)	-7 (-6.5)	-7 (-7.0)
REPLENISHMENT	1 (.3)	-1 (- .5)	1 (.5)	11 (6.0)	0 (- .2)
TOTAL	1 (.1)	10 (1.4)	6 (.8)	42 (5.8)	19 (2.6)

Note: Table IV shows the absolute change in the cost components over optimal values with percent changes in parentheses.

TABLE V

Average Costs Per Period for Multi-Item Systems:
Comparison of Statistical Control With
Optimal Control Given Full Information
(Revision Interval 24 Periods, Revision History Length 24 Periods)

COST COMPONENT	STATIONARY MODEL	MODEL I	MODEL II	MODEL III	MODEL IV
INVENTORY	24 (5.4)	40 (9.2)	31 (7.2)	68 (15.8)	58 (13.4)
BACKLOG	28 (25.9)	24 (23.2)	26 (25.0)	27 (26.5)	20 (19.2)
REPLENISHMENT	2 (.9)	0 (- .2)	1 (.7)	12 (6.2)	1 (.3)
TOTAL	54 (7.1)	64 (8.6)	59 (7.9)	106 (14.8)	79 (10.6)

Note: Table V shows the absolute increases in the cost components over optimal values with percentage increases shown in parentheses.

TABLE VI

Average Retrospective Simulation Forecasts for a 72-Item Negative Binomial System Controlled with Statistical Information About Demand: Comparison with Corresponding Estimated Expected Values (Percentage Underestimates)

	VARIANCE/MEAN = 9		VARIANCE/MEAN = 3	
COST COMPONENT	Power Approximation	Normal Approximation	Power Approximation	Normal Approximation
(a) 13-period Revision Interval, 13-period Revision History, 13-period Forecast History:				
INVENTORY	2.2	2.8	*	1.5
BACKLOG	76.2	82.0	*	66.9
REPLENISHMENT	1.6	1.8	*	1.1
TOTAL	25.1	32.8	*	15.6
(b) 26-period Revision Interval, 26-period Revision History, 26-period Forecast History:				
INVENTORY	1.2	1.6	1.1	0.9
BACKLOG	59.3	64.2	37.7	47.1
REPLENISHMENT	1.2	1.2	0.6	0.6
TOTAL	17.1	22.7	7.5	9.3
(c) 52-period Revision Interval, 52-period Revision History, 52-period Forecast History:				
INVENTORY	0.7	0.8	*	0.5
BACKLOG	42.1	44.6	*	28.0
REPLENISHMENT	0.5	0.6	*	0.3
TOTAL	10.7	14.0	*	5.0

* : Not available.

2. Summary of Results (continued)

A pattern of satisfactory performance emerges for the nonstationary models. The greatest degradation occurs for Models III and IV which are the two models with the most extreme variations in demand distributions.

Forecasting Operating Characteristics. A major study was conducted to find good methods of forecasting operating characteristics of statistically-derived policies. Early efforts in this area focused on the technique of *retrospective simulation*. This method produces forecasts at each policy revision by computing the operating characteristics that have occurred if the new policy were used on a recently-realized sequence of demands. The technique is inherently biased because the same demands are used to construct the policy and to forecast its performance. Computer simulation was used to estimate the magnitude of the bias for a variety of system parameter settings. Results are summarized in Table VI where percentage forecasting underestimates are given for total cost and its components in several 72-item systems. Notice that inventory and replenishment costs are forecast quite accurately. For example, in the variance-to-mean 9 system utilizing the Power Approximation with a 26-period revision interval, both inventory and replenishment cost forecasts are 1.2% below the actual values. This accuracy is not exhibited by forecasts of backlog quantity and, resultingly, total cost. In the example discussed above, the backlog quantity forecast is 59.3% below the actual value and the total cost forecast is a 17.1% underestimate. The biases are especially large for high penalty cost items.

Attempts to improve upon the retrospective simulation method have focused in two major approaches. The first approach is to use demand statistics in previously-developed formulas for the operating characteristics. The second approach considers several variations of retrospective simulation in which distribution functions are fitted to the demand statistics and more sophisticated simulation is performed. Results indicate that only modest gains can be realized by these methods.

BIBLIOGRAPHY

(1) Veinott, A.F., Jr. and H.M. Wagner, "Computation of Optimal (s,S) Inventory Policies," Management Science, Vol. 11, 1965, pp. 525-552.

(2) Wagner, H.M., M.O'Hagan and B. Lundh, "An Empirical Study of Exactly and Approximately Optimal Inventory Policies," Management Science, Vol. 11, 1965, pp.690-723.